Loschmidt echo for a chaotic oscillator

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Chaotic dynamics of a nonlinear oscillator is considered in the semiclassical approximation. The Loschmidt echo as a measure of quantum stability to a time dependent variation is calculated. It is shown that an exponential decay of the Loschmidt echo is due to a Lyapunov exponent and it has a pure classical nature. The Lyapunov regime is observed for a time scale which is of the power law in semiclassical parameter.

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I. INTRODUCTION

Classical chaotic dynamics can be characterized by a Lyapunov exponent Λ. The quantized procedure stops the classical spread of stretching and folding due to the uncertainty principle and, as a result, breaks the applicability of the semiclassical approximation. The corresponding breaking time was found in Ref. [1] to be

$$\tau_\hbar = (1/\Lambda)\ln(I_0/\hbar), \tag{1}$$

where I_0 is a characteristic action. It indicates a fast (exponential) growth of quantum corrections to the classical dynamics due to chaos. Recently this result gained a renewed interest also related to the fidelity of wave functions or the Loschmidt echo. In the field of quantum chaos a question on stability of trajectories can be readdressed to stability of wave functions with respect to a small variation of a control parameter [3] called the fidelity of the wave functions, which is a measure of a quantum reversibility. It is also named "Loschmidt echo" [4], and it is known that on the time scale of the order of $\tau_\hbar$ it decays exponentially $\sim e^{-\Lambda t}$ [4]. This regime, known as a Lyapunov one, is a result of "hypersensitivity" of this time reversibility to perturbation [5]. This result stimulated extensive studies and different decays of the Loschmidt echo have been observed for integrable [6,8–10], chaotic [6,7,9,11–13] and quasi-integrable [14] systems (see also references therein). Among these results, general consideration of the Loschmidt echo beyond the Lyapunov regime has been presented: e.g., time crossover from the Lyapunov regime to the quantum one with the Gaussian decay [6,7] has been observed [11] on the Heisenberg time scale of the order of $1/\hbar$; the Fermi golden rule decay was observed versus the Lyapunov regime [7]; also several regimes of fidelity decay have been clarified with a presentation of the semiclassical description for both chaotic and integrable cases [6,8]; as well as the following development of a general approach to echo [15] and consideration of unbounded systems like a Lorentz gas [16,17] have been presented.

In the present paper, we show that the Loschmidt echo in the nonlinear kicked oscillator decays exponentially due to the Lyapunov exponent in the Lyapunov regime [4,13]. An analytical expression for this behavior in the framework of the semiclassical expansion is obtained. For an experimental setup, this Loschmidt echo for the nonlinear oscillator is also addressing the possible application for echo spectroscopy in quantum optics [18].

Dynamics of a nonlinear oscillator with the Hamiltonian

$$\mathcal{H}_0 = \hbar\omega a^\dagger a + \hbar^2\mu(a^\dagger a)^2 \tag{2}$$

is integrable [19]. Here ω and μ are linear frequency and nonlinearity parameters correspondingly, while, for the chosen notation, annihilation and creation operators have the commutation rule $[a,a^\dagger]=1$. There is nontrivial semiclassical expansion that leads to an appearance of so-called D forms. These forms are determined as derivatives over the initial conditions, say, (α,α^*) as

$$D \equiv D(A,B) = \left(\frac{\partial A}{\partial\alpha}\right)\left(\frac{\partial B}{\partial\alpha^*}\right), \tag{3}$$

where $A(B)=\langle\psi(\alpha)|\hat{A}(\hat{B})|\psi(\alpha)\rangle$ is the average value of the operator. Therefore, the D forms determine the local instability of a dynamical system. In the presence of a perturbation dynamics becomes chaotic and the exponential growth of the D forms due to the Lyapunov exponents leads to the logarithmic breaking time of Eq. (1), known also as the Ehrenfest time. That result could be transparently seen in the coherent state basis [1,2,19], and it is independent of the choice of the initial basis of wave functions [20]. Role of the D forms as a sensitivity of dynamical variables to the initial conditions or to the perturbation can be also readdressed to the wave functions. More detailed consideration on the D forms for different dynamical realizations in the nonlinear oscillator is considered separately elsewhere [21].

Originally, a question on sensitivity of wave functions to a variance was asked in Ref. [3] to characterize quantum chaos by the fidelity of the wave function

$$M(t) = \left|\langle\psi_0|\exp\left\{i\int(\mathcal{H}+\delta\mathcal{H})dt/\hbar\right\}\exp\left\{-i\int\mathcal{H}dt/\hbar\right\}\times|\psi_0\rangle\right|^2. \tag{4}$$

It characterizes an evolution of the initial wave function ψ_0 governed by the two slightly different Hamiltonians $\mathcal{H}$ and the variational Hamiltonian $\mathcal{H}+\delta\mathcal{H}$. The fidelity $M(t)$ was also referred to as the "Loschmidt echo" [4], where dynamics of the initial wave function due to $\mathcal{H}$ after time t is reversed back to the initial state with the variational Hamiltonian $\mathcal{H}$

$+\delta\mathcal{H}$. The dynamical decay of the overlap is characterized by the Lyapunov exponent on some semiclassical time scale $\tau_{sc\ell}$ that also stands to be determined.

II. EVOLUTION OF A WAVE FUNCTION

We will consider analytically the overlap function $M(t)$ for chaotic dynamics of the nonlinear oscillator (2) in the presence of a periodic perturbation of the form of δ kicks with a period T and an amplitude ϵ. The Hamiltonian of the system is

$$\mathcal{H} = \hbar\omega a^\dagger a + \hbar^2\mu(a^\dagger a)^2 - \hbar\epsilon(a^\dagger + a)\sum_{n=-\infty}^{\infty}\delta(t-nT) \equiv \mathcal{H}_0 + V. \quad (5)$$

For the initial wave function we choose the coherent state. At the moment $t=0$ it is determined as an eigenfunction of the annihilation operator: $a(t=0)|\alpha(t=0)\rangle \equiv a|\alpha\rangle = \alpha|\alpha\rangle$. It enables one to study its dynamical evolution analytically in the semiclassical limit. Evolution of a generating function was considered in detail in Ref. [22]. We borrow some details of that analysis with corresponding adaptation to the present consideration. The evolution operator is

$$U(t) = \widehat{\exp}\left\{-i\int_0^t d\tau[\omega a^\dagger a + \hbar\mu(a^\dagger a)^2 - \epsilon g(\tau)(a^\dagger + a)]\right\}, \quad (6)$$

where $\widehat{\exp}$ means T-ordered exponential, while the perturbation is written as $V=-\hbar\epsilon g(\tau)(a^\dagger + a)$. Under this T-ordering symbol all exponents commute. Therefore, one can use the following Stratonovich-Hubbard transform [24] for the exponential

$$\widehat{\exp}\left[-i\hbar\mu T\int_0^t d\tau(a^\dagger a)^2/T\right] = \int\prod_\tau \frac{d\lambda(\tau)}{\sqrt{4\pi i\kappa}} \times\exp\left[i\int_0^t d\tau\lambda^2(\tau)/4\kappa\right]\widehat{\exp}\left[-i\int_0^t d\tau\lambda(\tau)a^\dagger a\right], \quad (7)$$

where we use that $\kappa=\hbar\mu T$ and $t/T\to t$ is a number of kicks represented in the continuous form. The auxiliary field $\lambda(\tau)$ is a quasi-random one with the imaginary Gaussian distribution, and the integration over $\lambda(\tau)$ in (7) corresponds formally to the many-dimensional Fourier transform. We take into account that the harmonic oscillator, acting on the coherent state, changes its phase only, and the perturbation V acts as a shift operator. Namely, e.g., for the period one writes

$$e^{i\epsilon(a^\dagger+a)}e^{-i\phi_\lambda(1)a^\dagger a}|\alpha\rangle = \exp[i(\epsilon/2)(\alpha e^{-i\phi_\lambda(1)} + \alpha * e^{i\phi_\lambda(1)})] \times|\alpha e^{-i\phi_\lambda(1)a^\dagger a} + i\epsilon\rangle,$$

where $\phi_\lambda(1)=\int_0^1 d\tau[\omega T+\lambda(\tau)]$ is a phase on the period 1. Therefore, acting by the unitary evolution operator $U(t=1)$ t times on $|\alpha\rangle$, we obtain finally that the wave function at time t has the form of the following functional integral [22]

$$\Psi(t) = U(t)|\alpha\rangle = \int\prod_\tau\left[d\lambda(\tau)/\sqrt{4\pi i\kappa}\right]\exp\left[i\int_0^t d\tau\lambda^2(\tau)/4\kappa\right] \times\exp[i\epsilon\int_0^t d\tau g(\tau)[\alpha_\lambda^*(\tau) + \alpha_\lambda(\tau)]/2]|\alpha_\lambda(t)\rangle, \quad (8)$$

where

$$\alpha(t) = e^{-i\phi_\lambda(t)}a(t) = e^{-i\phi_\lambda(t)}\left[\alpha + i\epsilon\int_0^t d\tau g(\tau)e^{i\phi_\lambda(\tau)}\right], \quad (9)$$

$$\phi_\lambda(t) = \int_0^t d\tau[\omega T + \lambda(\tau)]. \quad (10)$$

The wave function (8) is the evolution of the initial ket vector $|\alpha\rangle$ due to the Hamiltonian $\mathcal{H}$ of (5). To obtain an echo, we reverse dynamics at the moment t back to $t=0$ with a random (e.g., Gaussian) time-dependent process $\eta(t)$ to add it to the linear frequency ω. This variation of the linear frequency affects efficiently the chaos control parameter K, since the last is $K=4\epsilon\mu T|\alpha(t)|^2$ [1]. In this context, this approach is relevant to Refs. [10,13]. It should be admitted here that though the variation $\eta(t)$ is a time-dependent process, it is also relevant to the "traditional" fidelity problem [4], where the variation is static. For instance, it could be quasistatic variation, which is a constant shift for the linear frequency for the same period T, and different for the deferent periods. Therefore, evolution of the initial bra vector $\langle\alpha|$ is due to the variational Hamiltonian [23] $\mathcal{H}+\delta\mathcal{H}$, with the random time-dependent frequency

$$\omega \to \omega_\eta = \omega + \eta(t)/T. \quad (11)$$

The wave function of the variational motion is

$$\Psi_\eta^*(t) = [U(\eta,t)|\alpha\rangle]^\dagger. \quad (12)$$

For simplicity, we consider the Loschmidt echo averaged over the Gaussian distribution

$$P[\eta(t)] \equiv P(\eta(t),\overline{\eta},\sigma) = \frac{1}{\sqrt{4\pi\sigma}}\exp\left[-\int_0^t d\tau[\eta(\tau) - \overline{\eta}]^2/4\sigma\right] \quad (13)$$

with the nonzero first moment $\overline{\eta}$ and the variance σ. It reads

$$M_\sigma(t) = \int\prod_\tau d\eta(\tau)P[\eta(\tau)]M(\eta,t), \quad (14)$$

where

$$M(\eta,t) = |\langle\Psi_\eta(t)|\Psi(t)\rangle|^2 \quad (15)$$

is the Loschmidt echo for a one fixed realization of $\eta(\tau)$ with $0<\tau<t$. Denoting by

$$\beta_\lambda = -i\int_0^t d\tau g(\tau)\alpha_\lambda(\tau),$$

we obtain the scalar product of the wave function in Eq. (15) in the form

$$\mathcal{M}(\eta,t) \equiv \mathcal{M} = \langle \Psi_\eta(t)|\Psi(t)\rangle = \int \prod_\tau \frac{d\lambda_1(\tau)d\lambda_2(\tau)}{4\pi\kappa} \times \exp\left[\frac{i}{4\kappa}\int_0^t d\tau[\lambda_1^2(\tau)-\lambda_2^2(\tau)]\right] \times \exp\left[iIm(\alpha^*_{\lambda_2}\alpha_{\lambda_1} - \epsilon\beta_{\lambda_1} - \epsilon\beta^*_{\lambda_2}) - \frac{1}{2}|\alpha_{\lambda_1}-\alpha_{\lambda_2}|\right], \quad (16)$$

where $\alpha_{\lambda_1}, \alpha^*_{\lambda_2}, \beta_{\lambda_1}, \beta^*_{\lambda_2}$ depend on $\eta(\tau)$ due to the shift of the linear frequency ω in Eqs. (10) and (11).

III. SEMICLASSICAL APPROXIMATION

In the semiclassical limit, when $\kappa \ll 1$, the functional integral in Eq. (16) is strongly simplified, and its evaluation is analytically tractable. After linear change of variables

$$\lambda_1 = 2\mu + \kappa\nu/2, \quad \lambda_2 = 2\mu - \kappa\nu/2 - \eta, \quad (17)$$

the phases are

$$\phi_{\lambda_1} = \int d\tau[\omega T + \lambda_1] = \int d\tau[\omega T + 2\mu + \kappa\nu/2],$$

$$\phi_{\lambda_2} = \int d\tau[\omega T + \lambda_2 + \eta] = \int d\tau[\omega T - 2\mu + \kappa\nu/2].$$

The Jacobian of the transformation is 2κ. Taking into account these expressions for the phases we obtain from Eqs. (9) and (10) in the semiclassical approximation that

$$\alpha^*_{\lambda_2}(t)\alpha_{\lambda_1}(t) - \epsilon\beta_{\lambda_1} - \epsilon\beta^*_{\lambda_2} \approx -i\kappa\int_0^t d\tau\nu(\tau)|a(\tau)|^2, \quad (18)$$

$$e^{i\phi_{2\mu}(t)}[\alpha_{\lambda_1} - \alpha_{\lambda_2}] \approx -i\kappa\int_0^t d\tau\nu(\tau)a(\tau). \quad (19)$$

Here $a(\tau)$, defined in Eq. (9), is taken for $\nu(\tau)\equiv 0$. It gives the definition of the classical action as $I(t)=\kappa|a(t)|^2$ for $\kappa \to 0$ and $|a(t)|^2 \to \infty$. To carry out integration over $\nu(\tau)$ we use the following auxiliary expression [22]:

$$\exp\left[-(\kappa^2/2)\left|\int_0^t d\tau\nu(\tau)a(\tau)\right|^2\right] = \frac{2}{\pi\kappa}\int d^2\xi e^{-2|\xi|^2/\kappa} \times \exp\left[-i\frac{\sqrt{\kappa}}{2}\,\mathrm{Re}\,\xi^*\int_0^t d\tau\nu(\tau)a(\tau)\right] \quad (20)$$

Substituting Eqs. (18)–(20) into Eq. (16), we obtain for the scalar product of the wave functions

$$\mathcal{M} = \frac{2}{\kappa\pi}\int d^2\xi e^{-2|\xi|^2/\kappa}\int\prod_\tau \frac{d\mu(\tau)d\nu(\tau)}{2\pi} \times \exp\left[\frac{-i}{4\kappa}\int d\tau[(4\mu(\tau)-\eta(\tau)][\kappa\nu(\tau)-\eta(\tau))]\right] \times \exp\left[i\kappa\int d\tau\nu(\tau)[|a(\tau)+\xi|^2 - |\xi|^2]\right]. \quad (21)$$

The functional integrations over $\nu(\tau)$ is exact and gives the δ function in μ. Hence the integration over $\mu(\tau)$ is also exact. After these integrations we obtain from Eq. (21)

$$\mathcal{M} = \frac{2}{\kappa\pi}\int d^2\xi e^{-2|\xi|^2/\kappa}\exp\left[\frac{i}{\kappa}\int d\tau\eta(\tau)\overline{I_{c\ell}}(\alpha,\alpha^*,\tau)\right], \quad (22)$$

where $\overline{I_{c\ell}}(\alpha,\alpha^*,\tau) \equiv I(\omega T - 2|\xi|^2, \alpha+\xi, \alpha^*+\xi^*, \tau)$, and we neglect a small term of the order of η^2 in the exponential. Let us expand the last exponential in Eq. (22) in the Taylor series in ξ and ξ^*. Therefore, we have

$$\exp\left[\frac{i}{\kappa}\int d\tau\eta(\tau)\overline{I_{c\ell}}(\alpha,\alpha^*,\tau)\right] \equiv \mathcal{F}(\omega T - 2|\xi|^2, \alpha+\xi, \alpha^* + \xi^*) = \sum_{m,n,\ell}\frac{1}{n!\,m!\,\ell!}\frac{\partial^m}{\partial\alpha^m}\frac{\partial^n}{\partial\alpha^{*n}}\frac{\partial^\ell}{\partial(\omega T)^\ell} \times \mathcal{F}(\omega T,\alpha,\alpha^*)\xi^m\xi^{*n}(-2|\xi|^2)^\ell. \quad (23)$$

Substituting Eq. (23) in Eq. (22) and taking into account that

$$\frac{2}{\pi\kappa}\int d^2\xi e^{-2|\xi|^2/\kappa}\xi^p\xi^{*q} = \sqrt{(\kappa/2)^{p+q}}\sqrt{p!\,q!}\,\delta_{p,q},$$

we obtain an expression for the scalar product in the form of the expansion in the semiclassical parameter κ

$$\mathcal{M} = \sum_{n,\ell}\frac{(n+\ell)!}{n!\,n!\,\ell!}\frac{\partial^{2n}}{\partial\alpha^n\partial\alpha^{*n}}\frac{\partial^\ell}{\partial(\omega T)^\ell}\mathcal{F}(\omega T,\alpha,\alpha^*) \times(-2)^\ell(\kappa/2)^{n+\ell}. \quad (24)$$

It should be stressed that the strongest contribution to the sum (24) for the same orders of κ is due to the derivatives over the initial conditions, namely due to the D form

$$D(\tau) \equiv D(I,I) = \frac{1}{\kappa}\left(\frac{\partial I(\tau,\alpha,\alpha^*)}{\partial\alpha}\right)\left(\frac{\partial I(\tau,\alpha,\alpha^*)}{\partial\alpha^*}\right) \propto e^{2\Lambda\tau}. \quad (25)$$

Therefore, we obtain, approximately

$$\mathcal{M}_\eta(t) \approx \exp\left[\frac{i}{\kappa}\int d\tau\eta(\tau)I_{c\ell}(\tau)\right] \times\exp\left[-\int d\tau\eta(\tau)\int d\tau'\,\eta(\tau')D[I_{c\ell}(\tau),I_{c\ell}(\tau')]\right]. \quad (26)$$

Finally, using this approximation, we obtain that the

Loschmidt echo is determined by the following Gaussian integral:

$$M_\sigma(t) \propto \int \prod_\tau d\eta(\tau) P[\eta(\tau)] |\mathcal{M}(\eta,t)|^2. \quad (27)$$

In the case of narrow packet $\sigma \ll 1$, that might be roughly approximated by a δ-function $P[\eta(\tau)] \to \delta(\eta(\tau) - \overline{\eta})$, we obtain

$$M_\sigma(t) \propto \exp\left[-\frac{\overline{\eta}^2}{\kappa\Lambda^2} D(t)\right] \approx \exp\left[-\frac{\overline{\eta}^2}{\Lambda^2} e^{2\Lambda t}\right]. \quad (28)$$

A possibility of such super-exponential drop-off is also mentioned in Ref. [9]. For an arbitrary Gaussian distribution, we obtain that the Loschmidt echo decays exponentially like $\exp[-\Lambda t]$. For instance, in the opposite case, when $\sigma \gg 1$, using the Fourier transform

$$e^{-\kappa A^2(t)} = \int \frac{d\xi}{\sqrt{4\pi}} e^{-\xi^2/4} e^{-i\xi A(t)},$$

where $A(t) = \int_0^t d\tau e^{\Lambda\tau} \eta(\tau)$, we can calculate the functional integral over $\eta(\tau)$ and then over ξ. Finally, we have

$$M_\sigma(t) = \left[2\pi + \frac{4\pi\sigma}{\Lambda}(e^{2\Lambda t} - 1)\right]^{-1/2} \propto \sqrt{\frac{\Lambda}{\sigma}} e^{-\Lambda t}. \quad (29)$$

These decays of $M_\sigma(t)$ in Eqs. (28) and (29) are determined by the classical Lyapunov exponent Λ. This result has pure classical nature and is independent of the semiclassical parameter κ, and it survives in the classical case when $\kappa = 0$, as well.

IV. SEMICLASSICAL TIME SCALE

An analytical evaluation of the Loschmidt echo $M(t)$ by means of the semiclassical expansion leads to some restriction on time of the validity of the semiclassical description. This time is definitely not coincided with the quantum breaking time $\tau_\hbar$ in Eq. (1). We show that $M(t)$ has sense on this semiclassical time scale and $M(t)$ decays exponentially due to the D form according Eqs. (26) and (28).

The validity of Eqs. (28) and (29) for this time scale can be obtained from the semiclassical expansion (18) and (19) in exponential (16) that is the semiclassical expansion for the linear oscillator exposed to the quasi-random field λ. Therefore, the Jacobian of the Hamiltonian flow is

$$J = \det|\partial[\alpha(t), \alpha^*(t)] / \partial[\alpha, \alpha^*]|$$
$$\approx 1 - 2\kappa \sin\phi_\mu(t) \int \nu(\tau) d\tau + \kappa^2 \left[\int \nu(\tau) d\tau\right]^2. \quad (30)$$

It reflects that the Liouville theorem is not valid for the averages [19]. Since $\nu(\tau)$ is the quasi-random field with the complex Gaussian distribution defined at the Stratonovich-Hubbard transform in Eq. (7), we were able to evaluate the integral in Eq. (30) substituting a mean value of ν. The first moment equals zero: $\langle \nu(\tau) \rangle = 0$, that is why we use the second moment equaled $\langle \nu^2(\tau) \rangle = 16i/\kappa$. Substituting the square root from the modulus of $\langle \nu^2(\tau) \rangle$ in Eq. (30), we obtain for the Jacobian

$$J \approx 1 - 8\kappa^{1/2} t \sin\phi_\mu + 16\kappa t^2.$$

Therefore, the validity of the semiclassical expansion carried out in the exponential (16) and, consequently, validity of the exponential decay of $M(t)$ due to the Lyapunov exponent defined in Eq. (29) is determined by the following semiclassical time scale:

$$t < \tau_{sc\ell} \sim 1/4\kappa^{1/2}. \quad (31)$$

In the semiclassical limit, when $\kappa \ll 1$ the following inequality is definitely true: $\tau_{sc\ell} \gg \tau_\hbar$.

V. CONCLUSION

We presented an analytical evaluation of the Loschmidt echo $M(t)$ by means of the expansion over the semiclassical parameter $\kappa = \hbar\mu T$. We shown that $M(t)$ has sense on the semiclassical time scale of the order of the square root of the Heisenberg time $t < \tau_{sc\ell} \sim 1/\sqrt{\kappa}$ which is much longer than the Ehrenfest time: $\tau_{sc\ell} \gg \tau_\hbar$. On this time scale $M(t)$ decays exponentially due to the D form according to Eqs. (26), (28), and (29). This behavior has a pure classical nature and is determined by the Lyapunov exponent Λ for both the super-exponential decay of Eq. (28) and the exponential decay due to Eq. (29). It is the Lyapunov regime. It should be also admitted that the echo is due to the time-dependent variation $\delta\mathcal{H}(t)$ which could even be a random process $\eta(t)$ defined in Eqs. (11) and (13). To some extent, this consideration is relevant to numerical studies of the Loschmidt echo for a quantum kicked rotor [13]. In this connection, the expression (31) might also be a possible explanation of the observation in Ref. [13] of the exponential decay of $M(t)$ in the Lyapunov regime for a kicked rotor on times much longer than the Ehrenfest time $t \gg \tau_\hbar$.

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